FAU-TP3-01/3
hep-ph/yymmdd
4 June 2001

Domain-like Structures in the QCD Vacuum, Confinement and Chiral-Symmetry Breaking

Alex C. Kalloniatis [1]
Special Research Centre for the Subatomic Structure of Matter
University of Adelaide
South Australia 5005, Australia
and
Sergei N. Nedelko [2]
Institut für Theoretische Physik III Universität Erlangen - Nürnberg
Staudtstraße 7 D-91058 Erlangen, Germany
and Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia

Abstract

We discuss how the inclusion of singular gauge fields in the partition function for QCD can lead to a domain-like picture for the QCD vacuum by virtue of specific conditions on quantum fluctuations at the singularities. With a simplified model of hyperspherical domain regions with interiors of constant field strength we calculate the basic parameters of the QCD vacuum, the gluon condensate, topological susceptibility, string constant and quark condensate, and briefly discuss confinement of dynamical quarks and gluons.

Presented at the Workshop on Lepton Scattering, Hadrons and QCD,

Adelaide, 26 March - 6 April, 2001.

[1]akalloni@physics.adelaide.edu.au
[2]nedelko@theorie3.physik.uni-erlangen.de

1 Introduction

Singular gauge fields are most probably unavoidable in nonabelian gauge theories [1]. In the last two decades great effort has gone into clarifying whether or not (and, if yes, then how) such fields provide for confinement and chiral symmetry breaking.

We address this question in a model based on an assumption that singularities in vector potentials are concentrated on hypersurfaces ∂V_j in Euclidean space, in the vicinity of which gauge fields can be divided into a sum of a singular pure gauge S_μ and a regular fluctuation part Q_μ, and where a certain colour vector n_j^a can be associated with ∂V_j. For such configurations not to have infinite action[2] the fluctuation fields charged with respect to n_j must obey specific boundary conditions on ∂V_j. The interiors of these regions thus constitute "domains" V_j. Gauge field modes neutral with respect to n_j^a are not restricted and provide for interactions between domains. In a given domain V_j the effect of fluctuations in neighbouring regions can be seen as an external gauge field $B^a_{j\mu}$ neutral with respect to n_j^a. This enables an approximation in which different domains are assumed to be decoupled from each other but, to compensate, a certain mean field is introduced in domain interiors.

Decomposing a general gauge field $A^j_\mu = S^j_\mu + Q^j_\mu$ and demanding finiteness of the classical action we come to the conditions

$$\breve{n}_j Q^{(j)}_\mu = 0,\ \psi = -i\,\not{\eta}^j e^{i\alpha_j\gamma_5}\psi,\ \bar{\psi} = \bar{\psi} i\,\not{\eta}^j e^{-i\alpha_j\gamma_5},\ x\in\partial V_j, \tag{1}$$

with the adjoint matrix $\breve{n}_j = T^a n_j^a$ appearing in the condition for gluons, and a bag-like boundary condition arising for quarks, with a unit vector $\eta^j_\mu(x) = x_\mu/|x|$ normal to ∂V_j.

To make the model analytically tractable we consider spherical domains with fixed radius R and approximate the mean field in V_j by a covariantly constant (anti-)self-dual configuration with the field strength

$$\hat{\mathcal{B}}^{(j)a}_{\mu\nu} = \hat{n}^{(j)} B^{(j)}_{\mu\nu},\ \tilde{B}^{(j)}_{\mu\nu} = \pm B^{(j)}_{\mu\nu},\ B^{(j)}_{\mu\nu}B^{(j)}_{\rho\nu} = B^2\delta_{\mu\rho},\ B = \text{const},$$
$$\hat{n}^{(j)} = t^3\cos\xi_j + t^8\sin\xi_j\ \left(\xi_j \in \{\frac{\pi}{6}(2k+1),\ k=0,\dots,5\}\right),$$

where the parameter B is the same for all domains and the constant matrix $n_j^a t^a$ belongs to the Cartan subalgebra. It should be stressed that there is no source for this field on the boundary and therefore it should be treated as strictly homogeneous in all further calculations. The homogeneity itself appears here just as an approximation.

Thus the partition function we will deal with can be written

$$\begin{aligned}\mathcal{Z} &= \mathcal{N}\lim_{V,N\to\infty}\prod_{i=1}^{N}\int_V\frac{d^4z_i}{V}\int_\Sigma d\sigma_i\int_{\mathcal{F}^i_Q}\mathcal{D}Q^i\int_{\mathcal{F}^i_\psi}\mathcal{D}\psi_i\mathcal{D}\bar{\psi}_i\\ &\qquad \delta[D(\breve{\mathcal{B}}^{(i)})Q^{(i)}]\Delta_{\rm FP}[\breve{\mathcal{B}}^{(i)},Q^{(i)}]\exp\left\{-S^{\rm QCD}_{V_i}\left[Q^{(i)}+\mathcal{B}^{(i)},\psi^{(i)},\bar{\psi}^{(i)}\right]\right\},\end{aligned}$$

where the thermodynamic limit assumes $V, N \to \infty$ but with the density $v^{-1} = N/V$ taken fixed and finite. The measure of integration over parameters characterising domains is defined as

$$\int\limits_{\Sigma} d\sigma_i \ldots = \frac{1}{48\pi^2}\int\limits_0^{2\pi} d\alpha_i \int\limits_0^{2\pi} d\varphi_i \int_0^{\pi} d\theta_i \sin\theta_i$$

$$\times \int_0^{\pi} d\omega_i \sum_{k=0,1} \delta(\omega_i - \pi k) \int_0^{2\pi} d\xi_i \sum_{l=0,1,2}^{3,4,5} \delta(\xi_i - (2l+1)\pi/6)\ldots. \quad (2)$$

Here φ_i and θ_i are spherical angles of the chromomagnetic field, ω_i is the angle between the chromomagnetic and chromoelectric fields, ξ_i is the angle in the colour matrix $\hat{n}_i$, α_i is the chiral angle and z_i is the centre of the domain V_i.

This partition function describes a statistical system of density N/V composed of noninteracting extended domain-like structures, each of which is characterised by a set of internal parameters and whose internal dynamics are represented by the fluctuation fields.

2 Mean Field Correlators

In this model the connected $n-$point correlator

$$\langle B^{a_1}_{\mu_1\nu_1}(x_1)\ldots B^{a_n}_{\mu_n\nu_n}(x_n)\rangle = B^n t^{a_1\ldots a_n}_{\mu_1\nu_1,\ldots,\mu_n\nu_n}\Xi_n(x_1,\ldots,x_n),$$
$$t^{a_1\ldots a_n}_{\mu_1\nu_1,\ldots,\mu_n\nu_n} = \int d\sigma_j n^{(j)a_1}\ldots n^{(j)a_n} B^{(j)}_{\mu_1\nu_1}\ldots B^{(j)}_{\mu_n\nu_n},$$

of field strength tensors,

$$B^a_{\mu\nu}(x) = \sum_j^N n^{(j)a} B^{(j)}_{\mu\nu}\theta(1-(x-z_j)^2/R^2),$$

can be calculated explicitly using the measure, Eq. (2). Translation-invariant functions

$$\Xi_n(x_1,\ldots,x_n) = \frac{1}{v}\int d^4z\theta(1-(x_1-z)^2/R^2)\ldots\theta(1-(x_n-z)^2/R^2)$$

emerge and can be seen as the volume of the region of overlap of n hyperspheres of radius R and centres $(x_1,\ldots,x_n)$, normalised to the volume of a single hypersphere $v = \pi^2R^4/2$. The functions Ξ_n are continuous and vanish if $|x_i - x_j| \geq 2R$. Correlations in the background field have finite range $2R$. The Fourier transform of Ξ_n is then an entire analytical function and thus correlations do not have a particle interpretation. The statistical ensemble of background fields is not Gaussian since all connected correlators are independent of each other and cannot be reduced to the two-point correlations.

The simplest application of the above correlators gives a gluon condensate density which to this approximation is

$$g^2\langle F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)\rangle = 4B^2.$$

Another vacuum parameter which plays a significant role in the resolution of the $U_A(1)$ problem is the topological susceptibility [3]. To define this we consider first the topological charge density for the colour group $SU(3)$ in the lowest approximation

$$Q(x) = \frac{g^2}{32\pi^2}\tilde{F}^a_{\mu\nu}(x)F^a_{\mu\nu}(x) = \frac{B^2}{8\pi^2}\sum_{j=1}^{N}\theta[1-(x-z_j)^2/R^2]\cos\omega_j,$$

where $\omega_j \in \{0,\pi\}$ depends on the duality of the j-th domain. For a given field configuration the topological charge is additive

$$Q = \int_V d^4x Q(x) = q(N_+ - N_-),\ q = B^2R^4/16,\ -Nq \le Q \le Nq$$

where q is a 'unit' topological charge, namely the absolute value of the topological charge of a single domain, and N_+ (N_-) is the number of domains with (anti-)self-dual field, $N = N_+ + N_-$. The probability of finding the topological charge Q in a given configuration is defined by the distribution

$$\mathcal{P}_N(Q) = \frac{\mathcal{N}_N(Q)}{\mathcal{N}_N} = \frac{N!}{2^N\,(N/2 - Q/2q)!\,(N/2+Q/2q)!},$$

where $\mathcal{N}_N(Q)$ is the number of configurations with a given charge and $\mathcal{N}_N$ is the total number of configurations. The distribution is symmetric about $Q = 0$, where it has a maximum for N even. For N odd the maximum is at $Q = \pm q$. Averaged topological charge is zero.

The topological susceptibility χ is determined by the two-point correlator of topological charge density, which in the lowest approximation gives

$$\chi = \int d^4x\langle Q(x)Q(0)\rangle = \frac{B^4}{64\pi^4}\int d^4x\,\Xi_2(x) = \frac{B^4R^4}{128\pi^2}. \quad (3)$$

3 Area Law for the Wilson Loop

To zeroth order in fluctuations the Wilson loop is given by the integral

$$W(L) = \lim_{V,N\to\infty}\prod_{j=1}^{N}\int_V\frac{d^4z_j}{V}\int d\sigma_j\frac{1}{N_c}\mathrm{Tr}\exp\left\{i\int_{S_L}d\sigma_{\mu\nu}(x)\hat{B}_{\mu\nu}(x)\right\}.$$

Note that path ordering in our case is not necessary since the matrices $\hat{n}^k$ are assumed to be in the Cartan subalgebra. Computationally it is convenient to consider a circular

contour in the (x_3, x_4) plane of radius L with centre at the origin. To illustrate the steps in the calculation we consider here the case of colour group $SU(2)$, though the final result for $SU(3)$ will be quoted below. For colour $SU(2)$ we have $\hat{n}^k = \epsilon^k \tau_3,\ \epsilon^k = \pm 1$. The thermodynamic limit assumes that the density N/V is fixed. Calculation of the colour trace gives

$$W(L) = \lim_{V,N\to\infty} \left[\int_V \frac{d^4 z_j}{V} \int d\sigma_j \frac{1}{2} \left(e^{iB^j_{\mu\nu} J_{\mu\nu}(z_j)} + e^{-iB^j_{\mu\nu} J_{\mu\nu}(z_j)} \right) \right]^N,$$
$$J_{\mu\nu}(z_k) = \int_{S_L} d\sigma_{\mu\nu}(x) \theta(1 - (x - z_k)^2/R^2).$$

We have exploited the property that the integral above does not depend on the index j. As the contour of the Wilson loop is in the (x_3, x_4)-plane, the only nonzero components of $J_{\mu\nu}$ are

$$J_{34} = -J_{43} = \int_{S_L} dx_3 dx_4 \theta(1 - (x - z)^2/R^2),\ B_{\mu\nu} J_{\mu\nu} = 2BJ_{43} \cos\theta,$$

where θ is the angle between chromoelectric field and the third coordinate axis. After integrating over the spatial orientations of the vacuum field the Wilson loop takes the form

$$W(L) = \lim_{N,V\to\infty} \left[\frac{1}{V} \int_V dz \frac{\sin 2BJ_{43}(z)}{2BJ_{43}(z)} \right]^N.$$

Integrating over z and then taking the thermodynamic limit ($N \to \infty$, $v = V/N = \pi^2 R^4/2$), gives finally for a large Wilson loop $L \gg R$

$$W(L) = \lim_{N\to\infty} \left[1 - \frac{1}{N} U(L) \right]^N = e^{-U(L)},\ U(L) = \sigma \pi L^2 + O(L),$$
$$\sigma = Bf(BR^2),\ f(z) = \frac{2}{\pi z} \left(1 - \frac{1}{2\pi z} \int_0^{2\pi z} \frac{dx}{x} \sin x \right),$$

which displays an area law. For $SU(3)$ the only difference is in the function $f(z)$ which turns out to be:

$$f(z) = \frac{2}{3\pi z} \left(3 - \frac{\sqrt{3}}{2\pi z} \int_0^{2\pi z/\sqrt{3}} \frac{dx}{x} \sin x - \frac{2\sqrt{3}}{\pi z} \int_0^{\pi z/\sqrt{3}} \frac{dx}{x} \sin x \right).$$

The function f is positive for $z > 0$ and has a maximum for $z = 1.55$. We choose this maximum to estimate the model parameters by fitting the string constant to the lattice result,

$$\sqrt{B} = 947\text{MeV},\ R^{-1} = 760\text{MeV}, \tag{4}$$

and get for the gluonic parameters of the vacuum

$$\sqrt{\sigma} = 420\text{MeV}, \ \chi = (197\text{MeV})^4, \ \frac{\alpha_s}{\pi}\langle F^2\rangle = 0.081(\text{GeV})^4, \ q = 0.15. \tag{5}$$

If there were clear separation of the two scales characterising the system (namely if either $\sqrt{B}R \gg 1$ or $\sqrt{B}R \ll 1$) then an approximate treatment of Green's functions for the fluctuation fields would be possible. We observe that there is no separation: $\sqrt{B}R \approx 1$. Thus an approximation based on large or small domains is not justifiable.

Observe also that if B goes to zero then the string constant vanishes. This underscores the role of the gluon condensate in the confinement of static charges. On the other hand, if the number of domains is fixed and the thermodynamic limit is defined as $V, R \to \infty, N = \text{const.} < \infty$, namely if the domains are macroscopically large, then $W(L) = 1$, which indicates the absence of a linear potential between infinitely heavy charges in a purely homogeneous field.

Since we have exactly integrated over background fields the role of finite range of correlation functions is hidden in the above calculation. In order to see this role explicitly one would need to decompose the integrand into an infinite series and integrate term by term. At this step all correlation functions of the background field up to infinite order would be manifest. The arguments about the importance of a fast decay of correlators for confinement of static charges [4] would be seen to apply here via this representation.

4 Confinement of Fluctuation Fields

Confinement of the fluctuational fields can be studied via the analytical properties of their propagators. For the types of fields and boundary conditions we have described here, the propagators of fluctuations can be analytically calculated by reduction of the problems for both quarks and gluons to that of a charged scalar field. The scalar case is essentially equivalent to the problem of a four-dimensional harmonic oscillator with the orbital momentum coupled to the external field, and the general solution for these Green's functions can be found exactly by standard decomposition over hyperspherical harmonics.

Qualitatively it is clear that due to the boundary conditions we are dealing with, the $x-$space propagators of charged fields are defined in regions of finite support where they have integrable singularities so that their Fourier transforms are entire functions in the complex momentum plane. This can be treated as confinement of dynamical fields[5]. A known consequence of entire propagators is a Regge spectrum of relativistic bound states[6], which is physically appealing. A more expansive analysis of confinement in this model will be given elsewhere[7].

5 Quark Condensate

Due to averaging over self- and anti-self-dual configurations and all possible values of the angle α in the partition function chiral symmetry is not broken explicitly. However, as we show below, a nonzero quark condensate arises in the massless limit due to an interplay between the random distribution of domains with self- and anti-self-dual field and the boundary conditions with random value of the chirality violating angle α.

The quark propagator is defined by the equations

$$\begin{aligned}
&(i\not\partial - \frac{1}{2}\hat{n}\gamma_\mu B_{\mu\nu}x_\nu - m)S(x,y) = -\delta(x,y), \\
&i\not\eta(x)e^{i\alpha\gamma_5}S(x,y) = -S(x,y),\ (x-z)^2 = R^2, \\
&S(x,y)i\not\eta(y)e^{-i\alpha\gamma_5} = S(x,y),\ (y-z)^2 = R^2,
\end{aligned} \tag{6}$$

where $\eta_\mu(x) = (x-z)_\mu/|x-z|$. Substitution

$$\begin{aligned}
&S(x,y) = (i\not D + m)[P_\pm \mathcal{H}_0 + P_\mp O_+ \mathcal{H}_1 + P_\mp O_- \mathcal{H}_{-1}], \\
&O_+ = N_+\Sigma_+ + N_-\Sigma_-,\ O_- = N_+\Sigma_- + N_-\Sigma_+, \\
&N_\pm = \frac{1}{2}(1 \pm \hat{n}/|\hat{n}|),\ \Sigma_\pm = \frac{1}{2}(1 \pm \vec{\Sigma}\vec{B}/B),\ \hat{B} = |\hat{n}|B,
\end{aligned}$$

shows that scalar functions $\mathcal{H}_\zeta(x,y)$ should solve the equations:

$$(-D^2 + m^2 + 2\zeta\hat{B})\mathcal{H}_\zeta = \delta(x,y),$$

If we were to look for solutions vanishing at infinity then the Green's function $\mathcal{H}_{-1}$ ($\zeta = -1$) would be divergent in the massless limit due to zero modes of the Dirac operator. The bag-like boundary conditions remove zero eigenvalues from the spectrum, and the massless limit is regular.

In order to avoid cumbersome calculations and render the role of the former zero modes transparent, we turn to the particular choice $y = z = 0$ and calculate the value of the quark condensate at the centre of the domain. In this case $\mathcal{H}_\zeta$ are functions of x^2 only, and the general solutions for scalar Green's functions take the form ($\mu_\zeta = m^2/2B + \zeta$)

$$\mathcal{H}_\zeta = \Delta(x^2|\mu_\zeta) + C_\zeta\Phi(x^2|\mu_\zeta),\ \Phi(x^2|\mu) = e^{-Bx^2/4}M(1+\mu, 2, Bx^2/2).$$

Here $\Delta(x^2|\mu)$ is scalar propagator which vanishes at infinity with mass $2B\mu$, and Φ_ζ is a solution to the homogeneous equation regular at $x^2 = 0$, expressed in terms of the confluent hypergeometric function. The constants C_ζ can be fit to implement the boundary condition. The terms $m\mathcal{H}_0$ and $m\mathcal{H}_1$ vanish in the massless limit and do not contribute to the condensate. The nontrivial contribution comes from $m\mathcal{H}_{-1}$. The

bag-like conditions imply that on the boundary $\mathcal{H}_{-1}$ satisfies a mixed condition, where $f' = df/d|x|$ and the sign $(-)+$ corresponds to an (anti-)self-dual domain,

$$2e^{\mp i\alpha} m\mathcal{H}_{-1} = -2\mathcal{H}'_{-1} - \hat{B}R^2\mathcal{H}_{-1}.$$

which leads to the relations

$$\lim_{m\to 0} m\mathcal{H}_{-1} = \frac{e^{\pm i\alpha}}{2\pi^2 R^3} F(\hat{B}R^2/2) e^{-\hat{B}x^2/4},\ \mathrm{Tr}S(0,0) = \frac{e^{\pm i\alpha}}{2\pi^2 R^3}\sum_{|\hat{n}|} F(\hat{B}R^2/2),$$
$$F(z) = e^z - z - 1 + \frac{z^2}{4}\int_0^\infty \frac{dt e^{2t - z(\coth t - 1)/2}}{\sinh^2 t}(\coth t - 1).$$

Note that the term in the propagator with nonzero trace is proportional to the zero mode of the Dirac operator: $\not{D} P_{\mp} O_{-} \exp(-\hat{B}x^2/4) = 0$.

Averaging this result over the angle α and (anti-)self-dual configurations and taking into account the α-dependence of quark determinant [8] $\det S^{-1} \propto \exp\{\pm iq\alpha\}$, with q being unit topological charge, we get a finite result

$$\langle\bar{\psi}\psi\rangle = -\frac{q}{2\pi^2 R^3(1+q)}\sum_{|\hat{n}|} F(\hat{B}R^2/2).$$

For B and R fixed by the string constant as in Eq. (4) this is equal to

$$\langle\bar{\psi}\psi\rangle = -(228\mathrm{MeV})^3,$$

which indicates spontaneous breaking of chiral symmetry. Final conclusions require complete calculation including averaging over positions of domains.

Acknowledgments

ACK is supported by the Australian Research Council. We are grateful to Frieder Lenz for fruitful discussions and critical comments, as well as to Garii Efimov, Jan Pawlowski and Lorenz von Smekal for valuable communication.

References

[1] V. N. Gribov, *Nucl. Phys.* B **139**, 1 (1978); I. M. Singer, *Comm. Math. Phys.* **60**, 7 (1978).

[2] F. Lenz, O. Jahn, *Phys. Rev.* D **58**, 085006 (1998); F. Lenz, S. Worlen, hep-ph/0010099.

[3] R.J. Crewther, *Phys.Lett.* B **70**, 349 (1977); G. Veneziano, *Nucl. Phys.* B **159**, 213 (1979); E. Witten, *Nucl. Phys.* B **156**, 269 (1979).

[4] G.Dosch, Y.A. Simonov, *Phys. Lett.* B **205**, 339 (1988).

[5] H. Leutwyler, *Phys. Lett.* B **96**, 154 (1980).

[6] G.V. Efimov, S.N. Nedelko, *Phys. Rev.* D **51**, 174 (1995); G.V. Efimov, G. Ganbold, hep-ph/0103101.

[7] S.Nedelko, A.Kalloniatis, *in preparation.*

[8] S. Duerr, A. Wipf, *Nucl. Phys.* B **443**, 201 (1995).